Unconstrained Optimization

(1) In case of one variable

$$y = f(x)$$

For optimization $\Rightarrow$

F.O.C $\leftarrow \dfrac{dy}{dx} = 0 \Rightarrow \boxed{x = x^*}$

From this we know the value of the variable at which the function is optimized.

S.O.C $\Rightarrow \dfrac{d^2y}{dx^2} \gtrless 0$ at $x = x^*$

(a) $\dfrac{d^2y}{dx^2} > 0$ at $x = x^*$ then the function is minimum

(b) $\dfrac{d^2y}{dx^2} < 0$ at $x = x^*$ then the function is maximum

(c) $\dfrac{d^2y}{dx^2} = 0$ at $x = x^*$ then we can not say whether the function is max or min.

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(2)

In case of more than one variable

$y = f(x_1, x_2, \ldots, x_n)$

In particular ⇒

$y = f(x_1, x_2)$

FOC ⇒

$$\left.\begin{array}{l} \frac{\partial y}{\partial x_1} = 0 \\ \\ \frac{\partial y}{\partial x_2} = 0 \end{array}\right] \quad x_1^*, x_2^*$$

S.O.C ⇒

(a) $\frac{\partial^2 y}{\partial x_1^2} > 0$

$\frac{\partial^2 y}{\partial x_2^2} > 0$

$$\frac{\partial^2 y}{\partial x_1^2} \cdot \frac{\partial^2 y}{\partial x_2^2} > \frac{\partial^2 y}{(\partial x_1)(\partial x_2)} \cdot \frac{\partial^2 y}{(\partial x_2)(\partial x_1)}$$

⇒ $\boxed{y_{11} \cdot y_{22} > y_{12} \cdot y_{21}}$

At $x_1 = x_1^*$, $x_2 = x_2^*$ then the function is minimum.

(b) $$\frac{\partial^2 y}{\partial x_1^2} < 0$$

$$\frac{\partial^2 y}{\partial x_1^2} \cdot \frac{\partial^2 y}{\partial x_2^2} > \frac{\partial^2 y}{(\partial x_1)(\partial x_2)}$$

$$\frac{\partial^2 y}{\partial x_2^2} < 0$$

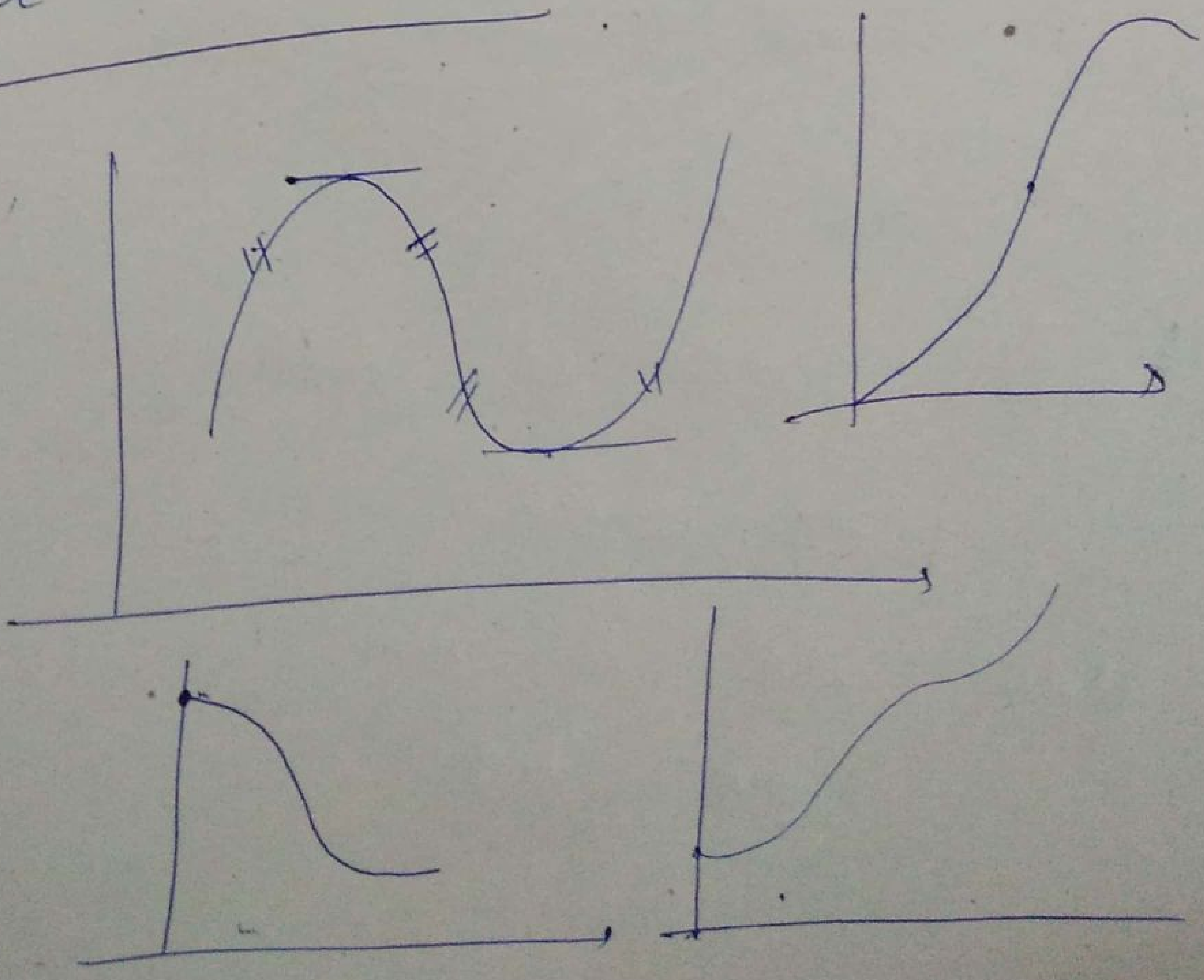
$$\frac{\partial^2 y}{(\partial x_2)(\partial x_1)}$$ → maxima

For inflexion pt

$$\frac{d^2 y}{dx^2} = 0 \Rightarrow \boxed{x = x^*}$$

$$\frac{d^3 y}{dx^3} \neq 0 \text{ at } x = x^*$$

The inflexion pt is that at which the curve change its curvature i.e from convex to concave or from concave to convex



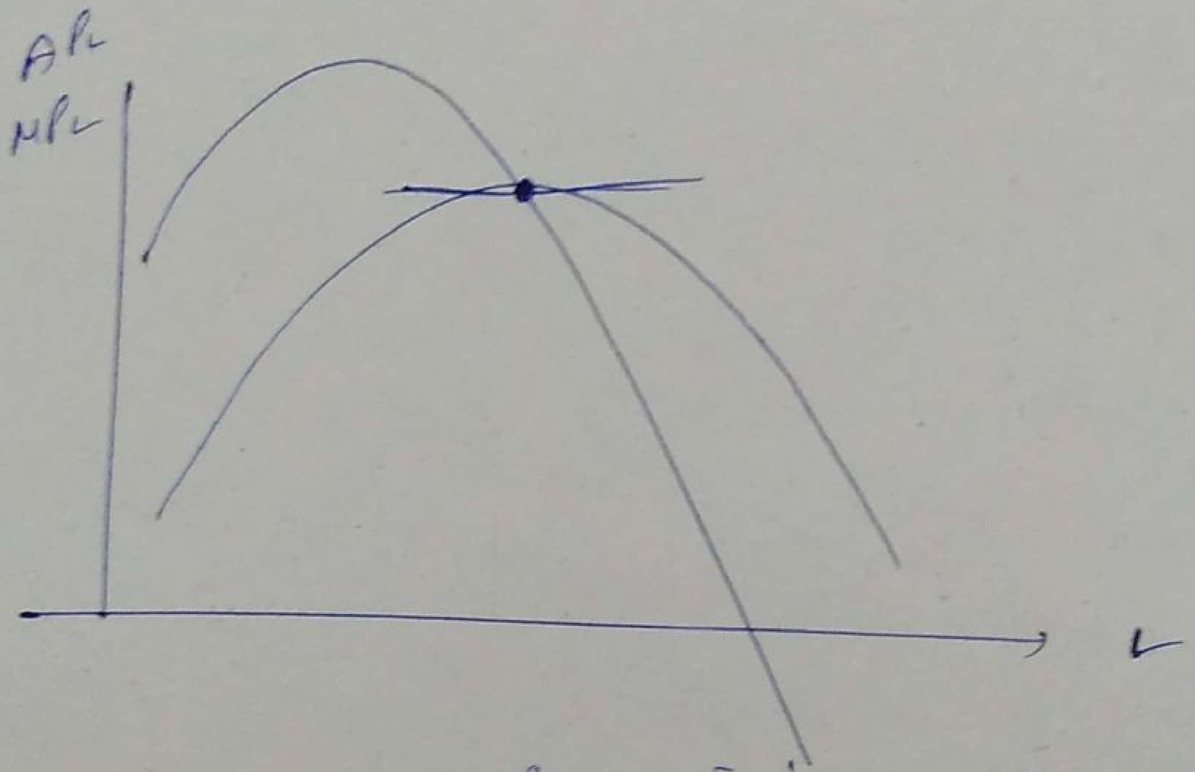
Application

(1) Total product, average product max

$$Q = f(L)$$

$$AP_L = Q/L$$

$$MP_L = f'(L)$$



when AP_L is rising then

$$AP_L < MP_L$$

when AP_L is falling

$$AP_L > MP_L$$

when AP_L is max

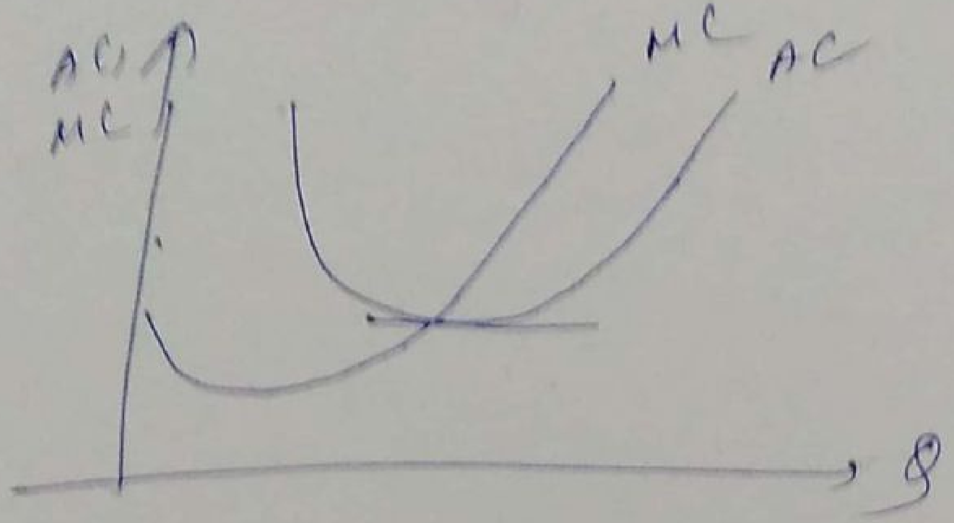
$$AP_L = MP_L$$

$MP_L = 0$ then TP is max

$\Rightarrow \boxed{L = L^*}$

At $L = L^*$ $\boxed{\frac{d^2Q}{dL^2} < 0}$

(8) cost minimisation $\Rightarrow$ AC and MC



$C = f(Q)$

$AC = C/Q = f(Q)/Q$

$MC = \frac{dC}{dQ}$

when AC is falling

AC > MC

when AC is rising

AC < MC

when AC is min

$AC = MC$

(3) Profit function

$$\pi = TR - TC$$

$$= R(q) - c(q)$$

F.O.C ⇒

$$\frac{d\pi}{dq} = R'(q) - c'(q) = 0$$

$$\Rightarrow \boxed{MR = MC} \Rightarrow q = q^*$$

$$\frac{d^2\pi}{dq^2} < 0 \Rightarrow \text{slope of } MR$$

$$< \text{slope of } MC$$

At $q = q^*$ the profit is maximum